

$\sigma_{DIS}(\nu N)$, NLO Perturbative QCD and $O(1 \text{ GeV})$ Mass Corrections

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The deep-inelastic neutrino-nucleon cross section is one of the components of few GeV neutrino interactions. We present here our results for neutrino-isoscalar nucleon charged current scattering including perturbative next-to-leading order QCD corrections, target mass corrections, charm mass and lepton mass corrections.

1. INTRODUCTION

Neutrino fluxes from cosmic ray interactions in the atmosphere, and neutrino beams from long-baseline neutrino oscillation experiments have in common that a range of processes contribute to the cross section and eventual event rates. In the few GeV region, neutrino quasi-elastic scattering [1,2], neutrino production of one pion or a few pions [3,4,5,6], and deep-inelastic scattering (DIS)[7,8] contribute to varying degrees [9]. Ultimately, one would like to reduce the theoretical uncertainties on the neutrino cross section in the few GeV region.

Discussed here are the contributions from charged current (CC) deep-inelastic scattering. Similar results are obtained for the neutral current case. We include lepton masses, the target mass M_N and charm mass corrections, all in the context of the next-to-leading order (NLO) QCD parton model [7,8].

2. FORMALISM

The differential cross section is

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dx dy} = \frac{G_F^2 M_N E_\nu}{\pi(1 + Q^2/M_W^2)^2}$$

$$\begin{aligned} & \times \left\{ \left(y^2 x + \frac{m_\ell^2 y}{2E_\nu M_N} \right) F_1^{TMC} \right. \\ & + \left[\left(1 - \frac{m_\ell^2}{4E_\nu^2} \right) - \left(1 + \frac{M_N x}{2E_\nu} \right) y \right] F_2^{TMC} \\ & \pm \left[xy \left(1 - \frac{y}{2} \right) - \frac{m_\ell^2 y}{4E_\nu M_N} \right] F_3^{TMC} \\ & + \frac{m_\ell^2 (m_\ell^2 + Q^2)}{4E_\nu^2 M_N^2 x} F_4^{TMC} \\ & \left. - \frac{m_\ell^2}{E_\nu M_N} F_5^{TMC} \right\}, \end{aligned} \quad (1)$$

where target (M_N) and charged lepton masses (m_ℓ) come into the limits of integration via

$$\frac{m_\ell^2}{2M_N(E_\nu - m_\ell)} \leq x \leq 1 \quad (2)$$

$$a - b \leq y \leq a + b \quad (3)$$

$$\begin{aligned} a &= \left[1 - m_\ell^2 \left(\frac{1}{2M_N E_\nu x} + \frac{1}{2E_\nu^2} \right) \right] \\ &\times (2 + M_N x/E_\nu)^{-1} \\ b &= \left[\left(1 - \frac{m_\ell^2}{2M_N E_\nu x} \right)^2 - \frac{m_\ell^2}{E_\nu^2} \right]^{1/2} \\ &\times (2 + M_N x/E_\nu)^{-1}. \end{aligned}$$

Target mass corrections (TMC) appear in the differential cross section (Eq. (1)), in the limits of integration, and implicitly in F_i . The structure functions depend on the Nachtmann variable [10]

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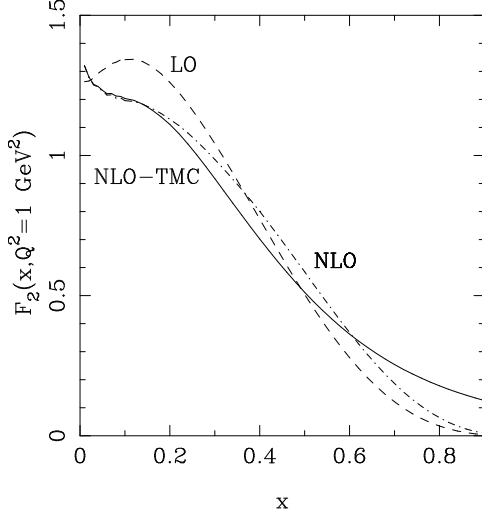


Figure 1. The neutrino isoscalar nucleon structure function F_2 plotted as a function of x for $Q^2 = 1 \text{ GeV}^2$. The solid line includes NLO QCD corrections with target mass corrections. With $M_N \rightarrow 0$, the dot-dashed line shows the NLO result and the dashed line is the LO result.

ξ defined by

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4M^2 x^2}{Q^2}}} . \quad (4)$$

The Nachtmann variable ξ reduces to x in the $M^2/Q^2 \rightarrow 0$ limit. In addition, one notes that the parton lightcone variable p^+ is a simple rescaling of the proton lightcone variable P^+ by $p^+ = \xi P^+$, however, $p^- \neq \xi P^-$ because $M_N \neq 0$. The result is that the hadronic tensor describing the weak interaction with the proton is not a simple rescaling of the partonic tensor [11]. This leads to M^2/Q^2 corrections. Further M^2/Q^2 corrections are attributed to the intrinsic transverse momentum of the partons [12]. Together, these can be derived via the operator product expansion [13,14] and lead to, for example, the target mass corrected structure function F_2^{TMC} in terms of the standard structure functions neglecting M_N :

$$F_2^{TMC} = 2 \frac{x^2}{\rho^3} \frac{\mathcal{F}_2(\xi, Q^2)}{\xi} \quad (5)$$

$$+ 12 \frac{M_N^2}{Q^2} \frac{x^3}{\rho^4} \int_{\xi}^1 d\xi' \frac{\mathcal{F}_2(\xi', Q^2)}{\xi'} \\ + 24 \frac{M_N^4}{Q^4} \frac{x^4}{\rho^5} \int_{\xi}^1 d\xi' \int_{\xi'}^1 \frac{\mathcal{F}_2(\xi'', Q^2)}{\xi''} ,$$

where

$$\rho^2 = 1 + \frac{4M^2 x^2}{Q^2} \quad (6)$$

$$\mathcal{F}_2(\xi, Q^2) = q(\xi, Q^2) + \bar{q}(\xi, Q^2) + \text{h.o.t.} \quad (7)$$

Similar equations obtain for the other structure functions, and are written in detail in Ref. [8].

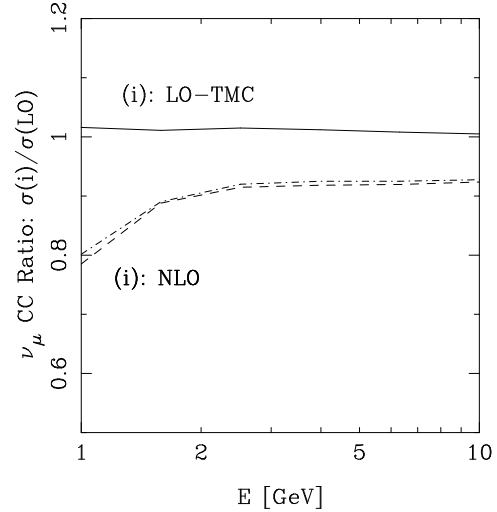


Figure 2. The ratio of the LO with TMC (solid line), NLO (dashed line) and NLO with TMC (dot-dashed line) to the LO cross section with no TMC, as a function of incident neutrino energy for ν_μ -isoscalar nucleon charged current scattering.

Charm quark mass corrections are incorporated via the ‘slow rescaling variable’ and charm mass corrected coefficient functions. Our results reported in Ref. [7] update those of Gottschalk [15], and they are consistent with Ref. [16] in the $M^2/Q^2 \rightarrow 0$ limit.

For the results here, we have added the additional constraint that the final state hadronic invariant mass be larger than a minimum value W_{min} in order to avoid the double counting issues associated with combining the cross sections for few pion processes and DIS. For results shown below, except as specified,

$$W^2 = Q^2 \left(\frac{1}{x} - 1 \right) + M_N^2 \geq W_{min}^2 = (1.4 \text{ GeV})^2. \quad (8)$$

We remark that the issue of combining DIS and exclusive processes is still an open question from a theoretical point of view, although various practical schemes are employed by a variety of Monte Carlo computer simulations.

We also note that we have included the full range of Q^2 in our evaluation of the cross sections, where the strong coupling constant and the parton distribution functions are frozen below a minimum value $Q^2 = (1.3 \text{ GeV})^2$. This extrapolation may or may not be justified for the low energy cross sections, introducing an additional theoretical uncertainty.

3. RESULTS

We first consider lepton mass corrections. For $\nu_\mu N \rightarrow \mu X$, the LO DIS cross section at 1 GeV is $\sim 60\%$ of the LO cross section with $m_\mu = 0$, denoted by σ_0 . By 2 GeV, the CC cross section is $\sim 95\%$ of σ_0 . By contrast, the CC cross section for $\nu_\tau N \rightarrow \tau X$ reaches 95% of σ_0 only at $E_\nu = 1 \text{ TeV}$.

To discuss target mass corrections, we keep $M_M \neq 0$ fixed in the expression for the differential cross section, Eq. (1). By neglecting the TMC, we set $\xi \rightarrow x$ and $F_i^{TMC}(\xi, Q^2)$ represented by equations like Eq. (5) are replaced by the uncorrected $F_i(x, Q^2)$. The result for neutrino structure function F_2^{TMC} for $Q^2 = 1 \text{ GeV}^2$ is shown in Fig. 1. The curves for LO, NLO and the target mass corrected NLO structure function are shown as a function of x , which is implicit in $\xi(x)$. The NLO-TMC curve lies below the LO curve at the smaller values of x , while the structure function with NLO and TMC is larger at large x . This comes from the fact that for $x = 1$ and $Q^2 = 1 \text{ GeV}^2$, $\xi \simeq 0.65$. The parton distribution functions are evaluated at $q(\xi, Q)$.

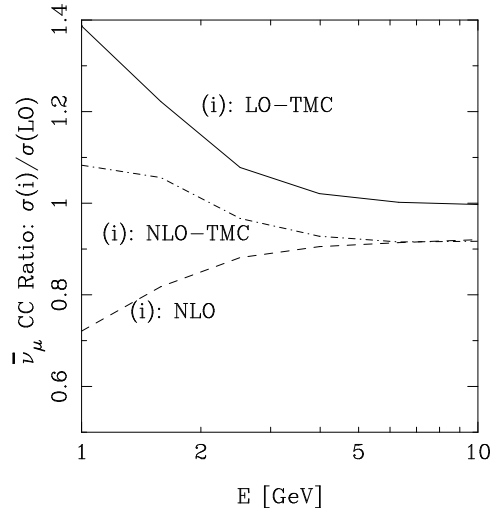


Figure 3. As in Fig. 2, for $\bar{\nu}_\mu$ -isoscalar nucleon charged current scattering.

In Fig. 2, the solid line shows the ratio of the leading order target mass corrected cross section to the leading order cross section with $M_N = 0$ as a function of incident muon neutrino energy. The next-to-leading order (NLO) ratio and NLO with target mass corrections ratio to the LO cross section is also shown in the figure. The target mass corrections are not very important for $\nu_\mu N$ scattering. A larger effect is seen in the case of $\bar{\nu}_\mu$ scattering with nucleons in Fig. 3. The target mass corrections tend to moderate the NLO corrections at low energies.

The different effects for neutrinos and antineutrinos can be attributed to the fact that the cross sections are dominated by the valence quarks. We note that

$$\frac{d\sigma}{dy}(\nu N) \sim q(x, Q^2) \cdot 1 \quad (9)$$

$$\frac{d\sigma}{dy}(\bar{\nu} N) \sim q(x, Q^2) \cdot (1 - y)^2. \quad (10)$$

The antineutrino cross section has a smaller average y and therefore a smaller Q^2 and larger M^2/Q^2 correction than the neutrino cross section. Figs. 4 and 5 show the same effect for ν_τ and $\bar{\nu}_\tau$ scattering with nucleons.

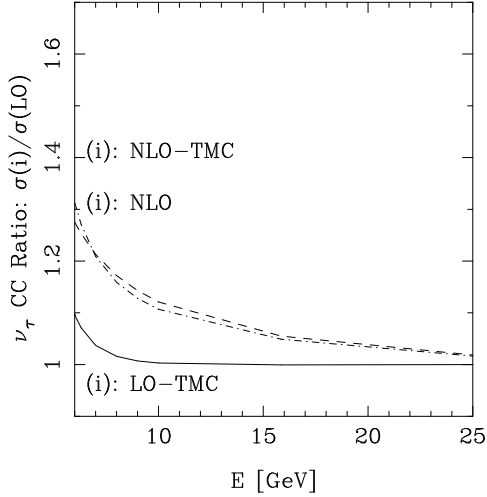


Figure 4. As in Fig. 2, for ν_τ -isoscalar nucleon charged current scattering.

The context of the NLO and target mass corrections can be seen from Figs. 6 and 7. In Fig. 6, we show that DIS contribution together with the quasi-elastic cross section and the cross section for Δ production. The quasi-elastic (QE) neutrino-isoscalar nucleon cross section is evaluated using the parameters summarized in Ref. [2] by Strumia and Vissani. The contribution coming from Δ production has been evaluated by a number of authors. Here, we use the recent results of Ref. [5], in which the cross section is evaluated for $W < 1.4$ GeV. The left-most solid curve labeled DIS is the LO evaluation of the DIS cross section for $W > 1.4$ GeV. The dashed line overlaying the solid line includes TMC. The dot-dashed line shows the NLO corrected cross section with $W > 1.4$ GeV. TMC to the NLO curve are indistinguishable from the NLO curve.

In addition to the DIS curve for $W > 1.4$ GeV, we also show for reference the DIS curves for $W > 1.6$ GeV and $W > 2.0$ GeV at leading order. These two curves give a hint to the uncertainty in the procedure for combining inclusive (DIS) with exclusive (QE and Δ) cross sections. These other two DIS cross sections should be combined with their respective Δ cross sec-

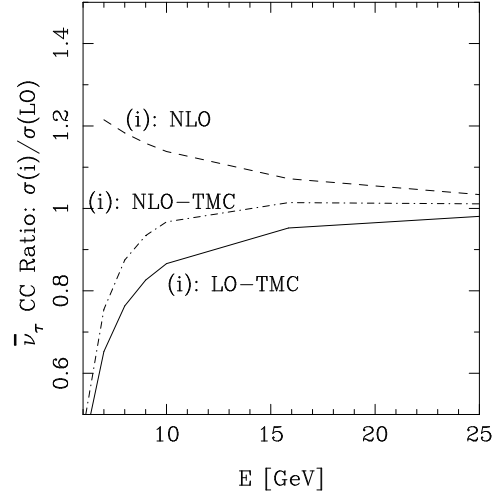


Figure 5. As in Fig. 2, for $\bar{\nu}_\tau$ -isoscalar nucleon charged current scattering.

tions with $W < 1.6, 2.0$ GeV to get the full inelastic cross sections.

The corresponding figure for ν_τ -isoscalar nucleon scattering is shown in Fig. 7. The production of Δ is much less significant in this case.

4. DISCUSSION

The range of DIS cross sections values in Figs. 6 and 7 as a function of the minimum value of W gives a hint of the range of uncertainty in the DIS contribution to the total neutrino cross section. Target mass corrections are not important on this scale of uncertainty, although they certainly are important in limited regions of phase space. Target mass corrections have a larger effect in antineutrino scattering in the lower energy range of interest for $\bar{\nu}_\mu$, and for a larger energy range for $\bar{\nu}_\tau$, partly compensating for the NLO enhancement of the LO cross section.

Another approach to evaluating the few GeV DIS cross section by Bodek and Yang [17] has been to phenomenologically model electron-hadron electromagnetic scattering in terms of rescaled parton distributions, then to use the appropriate parton distribution combinations for

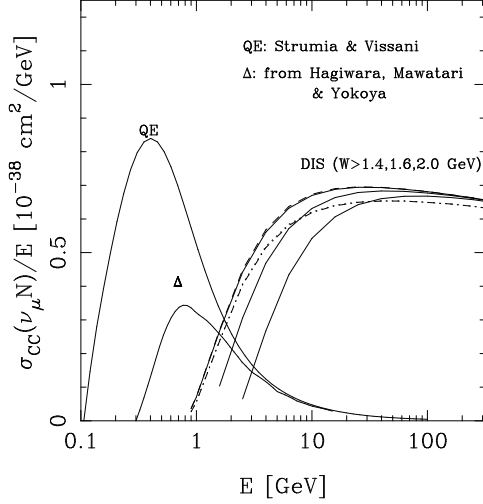


Figure 6. Quasi-elastic (QE), resonant Δ and deep inelastic contributions to $\nu_\mu N$ charged current scattering. The solid lines for the DIS contribution are LO, with $W > 1.4, 1.6$ and 2.0 GeV from left to right. The dot-dashed line is the NLO corrected DIS cross section with $W > 1.4$ GeV.

neutrino and antineutrino scattering with nucleons. A detailed comparison of this approach to the perturbative QCD evaluation with the order GeV mass corrections would be useful, in part to try to assess the importance of dynamical higher twist contributions to the cross section.

In principle, the transition from deep-inelastic to exclusive processes in neutrino scattering can be modeled using duality arguments [18], however, the details of a practical procedure are difficult to determine [19]. Our results here establish a reference standard for our knowledge of the perturbative QCD prediction for neutrino-nucleon inelastic scattering including lepton mass, charm mass and target mass corrections, an input to the theoretical study of the full picture of few GeV neutrino-nucleon interactions.

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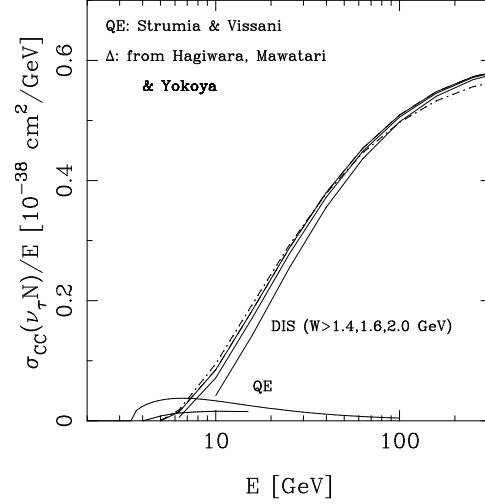


Figure 7. As in Fig. 6, for $\nu_\tau N$ charged current scattering.

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